HEAT AND MASS TRANSFER IN A LAYER OF POWDER MOVING OVER A HEATING SURFACE

V. V. Kornaraki UDC 536.24.001:66.047.75

A system of differential equations is given for heat and mass transfer, together with analytical relationships for the temperature, water content, drying rate, and heat-transfer rate for a packed layer moving in a slot channel.

An immobile bed with conductive heat transfer from a hot surface is used to dry many labile, explosive, and readily oxidized powders and finely divided materials for which direct contact with a drying agent is impermissible. However, the method is extremely inefficient, but a considerable acceleration can be attained by passing the material as a flowing layer over a heating surface [2]. Although this method is promising, no research on heat transfer in such layers has been done, and no mathematical description of the process has been published. Here we give a system of equations for the drying in such a moving layer, together with the solution for the particular case of a slot channel with unsymmetrical boundary conditions that reflects the actual setting in drying systems fairly closely.

We use the major concepts due to Lykov for drying in porous bodies [3]; as in [3-7], we assume a homogeneous model in which the moist granular material is considered as a continuous medium with effective transfer coefficients. The temperatures of all components (particles, liquid, and vapor) are considered as identical. It has been shown [1] that one is justified in using such a model for such a bed over a fairly wide range in the parameters. The following assumptions are made in deriving the equations: 1) The bed is in stable motion and moves as a rod; 2) the effective physical characteristics and the phase-transition criterion are constant; 3) the transfer of heat and mass by conduction along the flow is negligible in comparison with the convective component; 4) the mass transfer due to vapor infiltration is negligible, while the effects of the vapor infiltration on the heat transfer are incorporated into the effective thermal conductivity.

The system of differential equations then takes the dimensionless form

$$\frac{\partial T(Y,\ X)}{\partial X} = \frac{\partial^2 T(Y,\ X)}{\partial Y^2} - \varepsilon\,\mathrm{Ko}\,\frac{\partial \Theta(Y,\ X)}{\partial X};$$
$$\frac{\partial \Theta(Y,\ X)}{\partial X} = \mathrm{Ly}\,\frac{\partial^2 \Theta(Y,\ X)}{\partial Y^2} - \mathrm{Ly}\,\mathrm{Pn}\,\frac{\partial^2 \Theta(Y,\ X)}{\partial Y^2}. \qquad (1)$$

The equations for steady-state mass and heat transfer in a moving bed are similar to those for nonstationary conductive drying in an immobile bed [3, 7]; an analog of the Fourier criterion for the nonstationary case is the reduced channel length $X = (1/\mathrm{Pe})\cdot(x/D)$ in (1). System (1) was solved for a particular case to give the temperature and potential distributions, together with the heat-transfer and drying rates in a dryer consisting of a system of slot channels in which heat is brought up to the moving bed from a surface, while the resulting vapor is lost through slots.

The formation is as follows. The channel (Fig. 1) receives the moist material whose temperature t_0 and water content u_0 are constant throughout the cross section. The material moves in the channel as a dense layer with a speed v constant throughout the section and length. The left-hand wall is impermeable to water and vapor and supplies heat to the bed; this heat raises the temperature of the material, evaporates the water, and, in part, is lost via the right-hand permeable wall to the environment on account of convective heat and mass transfer. The process is described by (1) subject to the following boundary conditions:

M. V. Lomonosov Food-Technology Institute, Odessa. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 34, No. 6, pp. 1040-1047, June, 1978. Original article submitted April 22, 1977.

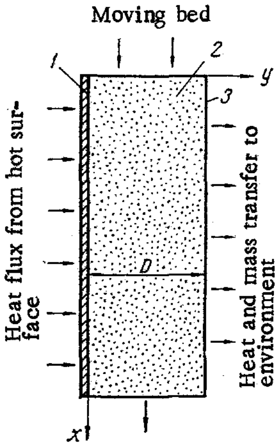

Fig. 1

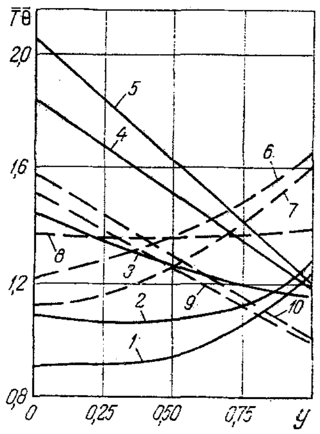

Fig. 2

Fig. 1. Drying in a moving bed with heat supplied from the surface: 1) heating surface; 2) moist material; 3) free (permeable) surface.

Fig. 2. Distributions of temperature (1-5) and potential (6-10) over the width of the channel: 1, 6) X = 0.1; 2, 7) 0.2; 3. 8) 0.4; 4, 9) 0.8; 5, 10) 1.6.

at the inlet,

$$T(Y,\ 0)=0;\quad \Theta(Y,\ 0)=0; \tag{2}$$

at the heating surface (conditions of the second kind),

$$\frac{\partial T(0,\ X)}{\partial Y}=-\mathrm{Ki}_q=\mathrm{const};\quad \frac{\partial\Theta(0,\ X)}{\partial Y}-\mathrm{Pn}\,\frac{\partial T(0,\ X)}{\partial Y}=0; \tag{3}$$

at the free surface (conditions of the third kind),

$$\frac{\partial T(1,\ X)}{\partial Y}-\mathrm{Bi}_q[1-T(1,\ X)]-(1-\varepsilon)\,\mathrm{Ko}\,\mathrm{Ly}\,\mathrm{Bi}_m[1-\Theta(1,\ X)]=0,$$

$$-\frac{\partial\Theta(1,\ X)}{\partial Y}+\mathrm{Pn}\,\frac{\partial T(1,\ X)}{\partial Y}+\mathrm{Bi}_m[1-\Theta(1,\ X)]=0. \tag{4}$$

The steady-state problem for a moving bed described by (1) with boundary conditions (2)-(4) is similar to the nonstationary problem for an immobile bed of [7].* The distributions of the temperature and potential in the moving bed are described by the following expressions, which are analogous to those of [7]:

$$T(Y,\ X)=1+\mathrm{Ki}_q\left(1+\frac{1}{\mathrm{Bi}_q}-Y\right)-\sum_{n=1}^{\infty}A_n\exp(-\mu_n^2X)\times$$
$$\times[(\nu_2^2-1)\,b_2(\mu_n)\cos(\nu_1\mu_nY)-(\nu_1^2-1)\,b_1(\mu_n)\cos(\nu_2\mu_nY)]; \tag{5}$$

$$\Theta(Y,\ X)=1-\mathrm{Pn}\,\mathrm{Ki}_q(1-Y)+\mathrm{Pn}\sum_{n=1}^{\infty}A_n\exp(-\mu_n^2X)\times$$
$$\times[b_2(\mu_n)\cos(\nu_1\mu_nY)-b_1(\mu_n)\cos(\nu_2\mu_nY)]. \tag{6}$$

The temperature and potential averaged over the cross section are

$$\bar T(X)=1+\mathrm{Ki}_q\left(\frac{1}{2}+\frac{1}{\mathrm{Bi}_q}\right)-\sum_{n=1}^{\infty}A_n\exp(-\mu_n^2X)\times$$
$$\times\left[(\nu_2^2-1)\,b_2(\mu_n)\,\frac{\sin(\nu_1\mu_n)}{\nu_1\mu_n}-(\nu_1^2-1)\,b_1(\mu_n)\,\frac{\sin(\nu_2\mu_n)}{\nu_2\mu_n}\right]; \tag{7}$$

*The nonstationary unsymmetrical case has previously been solved [4, 5] without allowance for the mass transfer arising from the potential gradient.

$$\bar{\Theta}(X) = 1 + \frac{\mathrm{Pn}\,\mathrm{Ki}_q}{2} + \mathrm{Pn}\sum_{n=1}^{\infty} A_n \exp(-\mu_n^2 X) \times$$

$$\times\left[b_2(\mu_n)\,\frac{\sin(\nu_1\mu_n)}{\nu_1\mu_n} - b_1(\mu_n)\,\frac{\sin(\nu_2\mu_n)}{\nu_2\mu_n}\right]. \tag{8}$$

The rates of change of the mean temperature and potential over the reduced length are

$$\frac{\partial \bar{T}}{\partial X} = \sum_{n=1}^{\infty} A_n\mu_n^2 \exp(-\mu_n^2 X)\left[(\nu_2^2-1)\,b_2(\mu_n)\,\frac{\sin(\nu_1\mu_n)}{\nu_1\mu_n} - (\nu_1^2-1)\,b_1(\mu_n)\,\frac{\sin(\nu_2\mu_n)}{\nu_2\mu_n}\right]; \tag{9}$$

$$\frac{\partial \bar{\Theta}}{\partial X} = -\,\mathrm{Pn}\sum_{n=1}^{\infty} A_n\mu_n^2 \exp(-\mu_n^2 X)\left[b_2(\mu_n)\,\frac{\sin(\nu_1\mu_n)}{\nu_1\mu_n} - b_1(\mu_n)\,\frac{\sin(\nu_2\mu_n)}{\nu_2\mu_n}\right]. \tag{10}$$

Then in (5)-(10) we have

$$\nu_1^2 = 0{,}5\left(1 + \varepsilon\,\mathrm{Ko}\,\mathrm{Pn} + \frac{1}{\mathrm{Ly}} - \sqrt{\left[\left(1 + \varepsilon\,\mathrm{Ko}\,\mathrm{Pn} + \frac{1}{\mathrm{Ly}}\right)^2 - \frac{4}{\mathrm{Ly}}\right]}\right);$$

$$\nu_2^2 = 0{,}5\left(1 + \varepsilon\,\mathrm{Ko}\,\mathrm{Pn} + \frac{1}{\mathrm{Ly}} + \sqrt{\left[\left(1 + \varepsilon\,\mathrm{Ko}\,\mathrm{Pn} + \frac{1}{\mathrm{Ly}}\right)^2 - \frac{4}{\mathrm{Ly}}\right]}\right);$$

$$A_n = \frac{2}{\mu_n^2}\left\{\left\{\frac{\mathrm{Ki}_q}{\nu_2^2-\nu_1^2}\,[b_2(\mu_n)\,c_2 - b_1(\mu_n)\,c_1] + \right.\right.$$

$$\left.\left. + \left\{\left[1 - (1-\varepsilon)\ \mathrm{Ko}\,\mathrm{Ly}\,\frac{\mathrm{Bi}_m}{\mathrm{Bi}_q}\right] b_1(\mu_n) + \frac{\nu_2^2-1}{\mathrm{Pn}}\,a_1(\mu_n)\right\} b_2(\mu_n)\right\}\right\} \times$$

$$\times\,\{b_2^2(\mu_n)\,c_2 d_1(\mu_n) - b_1^2(\mu_n)\,c_1 d_2(\mu_n)\}^{-1};$$

$$a_1(\mu_n) = \left[1 + (1-\nu_1^2)\,\frac{1-\varepsilon}{\varepsilon}\,\mathrm{Ly}\,\frac{\mathrm{Bi}_m}{\mathrm{Bi}_q}\right]\cos(\nu_1\mu_n) - \frac{\nu_1\mu_n}{\mathrm{Bi}_q}\sin(\nu_1\mu_n);$$

$$a_2(\mu_n) = \left[1 + (1-\nu_2^2)\,\frac{1-\varepsilon}{\varepsilon}\,\mathrm{Ly}\,\frac{\mathrm{Bi}_m}{\mathrm{Bi}_q}\right]\cos(\nu_2\mu_n) - \frac{\nu_2\mu_n}{\mathrm{Bi}_q}\sin(\nu_2\mu_n);$$

$$b_1(\mu_n) = \cos(\nu_1\mu_n) - \frac{\mu_n}{\mathrm{Ly}\,\mathrm{Bi}_m\nu_1}\sin(\nu_1\mu_n);$$

$$b_2(\mu_n) = \cos(\nu_2\mu_n) - \frac{\mu_n}{\mathrm{Ly}\,\mathrm{Bi}_m\nu_2}\sin(\nu_2\mu_n);$$

$$c_1 = (\nu_1^2 - 1)\left(\frac{1}{\mathrm{Bi}_q} - \frac{1}{\mathrm{Ly}\ \mathrm{Bi}_m}\right) - \frac{\mathrm{Ko}\,\mathrm{Pn}}{\mathrm{Bi}_q};$$

$$c_2 = (\nu_2^2 - 1)\left(\frac{1}{\mathrm{Bi}_q} - \frac{1}{\mathrm{Ly}\ \mathrm{Bi}_m}\right) - \frac{\mathrm{Ko}\,\mathrm{Pn}}{\mathrm{Bi}_q};$$

$$d_1(\mu_n) = 1 + \frac{\sin(\nu_1\mu_n)}{\nu_1\mu_n}\cos(\nu_1\mu_n);$$

$$d_2(\mu_n) = 1 + \frac{\sin(\nu_2\mu_n)}{\nu_2\mu_n}\cos(\nu_2\mu_n).$$

Here μ_n are the roots of the characteristic equation $f(\mu_n) = 0$, where

$$f(\mu_n) = (1-\nu_2^2)\,a_1(\mu_n)\,b_2(\mu_n) - (1-\nu_1^2)\,a_2(\mu_n)\,b_1(\mu_n).$$

Equations (5)-(10) allow one to evaluate the effects of the main factors on the temperature and potential distribution: bed speed, physical characteristics, channel geometry, transfer conditions, temperature and water content at the inlet, and phase-transition criterion. We see from (5)-(8) that the dimensionless local and mean temperatures and potentials increase from zero to their maximum value at $X = X_{st}$, beyond which the infinite sum can be neglected. Past this point, the temperature and potential are almost independent of the reduced length (and also of the bed speed), and a self-modeling situation occurs. Equations (5)-(8) take the following form for the region $X > X_{st}$:

$$T = 1 + \mathrm{Ki}_q\left(1 + \frac{1}{\mathrm{Bi}_q} - Y\right); \tag{5a}$$

$$\Theta = 1 + \mathrm{Pn}\,\mathrm{Ki}_q(1 - Y); \tag{6a}$$

$$\bar{T}_{\max} = 1 - \mathrm{Ki}_q\left(\frac{1}{2} - \frac{1}{\mathrm{Bi}_q}\right); \tag{7a}$$

$$\bar{\Theta}_{\max} = 1 + \frac{\mathrm{Pn}\,\mathrm{Ki}_q}{2}. \tag{8a}$$

The temperature and potential have linear distributions in this range; the bed acts as a planar wall and all the heat given out by the hot surface passes through the bed into the environment. The mean bed temperature is dependent on the heat-supply conditions, the heat transfer to the environment, and the thermophysical characteristics, while the mass-transfer characteristics have no effect, as (7a) shows. The maximum value for the mean potential (and, therefore, the minimal water content) is determined by the heat-supply conditions (and, therefore, the temperature of the bed) and by the mass-transfer characteristics, as (8) shows. No drying occurs in this region, and the bed acquires an equilibrium water content corresponding to the temperature (this may be less than the water content u_f corresponding to the environment, and then $\bar{\theta} > 1$).

The rates of change of the dimensionless temperature and potential decrease as the reduced length increases, and they tend to zero in the self-modeling region, as (9) and (10) show. Increase in the bed speed (Peclet number) causes the potential and water content to vary less rapidly over the length, but the total water loss increases on account of the increased flow of material.

From (10) we obtain the drying rate in the moving bed:

$$\frac{\partial \bar{u}}{\partial \tau} = \frac{(u_0 - u_f)\,a_q}{D^2}\ \mathrm{Pn} \sum_{n=1}^{\infty} A_n \mu_n^2 \exp(-\mu_n^2 X) \times$$
$$\times \left[b_2(\mu_n)\ \frac{\sin(\nu_1\mu_n)}{\nu_1\mu_n} - b_1(\mu_n)\ \frac{\sin(\nu_2\mu_n)}{\nu_2\mu_n} \right]. \tag{11}$$

We see from (11) that the drying rate is dependent on the reduced length of the channel and on the mass-transfer characteristics of the material; it decreases monotonically along the channel, but the more slowly, the higher the speed. The drying rate increases with the speed under otherwise equal conditions. We see from (11) that there is no region of constant drying rate in a moving bed. A simple exponential law applies for the drying rate if the reduced length is sufficiently large, and one need take only the first term in (11). The drying rate tends to zero as $X \to 0$.

The known temperature distribution gives us an expression for the heat-transfer rate at the heating surface:

$$\mathrm{Nu} = \mathrm{Ki}_q/(T_w - \bar{T}). \tag{12}$$

Here the local heat-transfer coefficient has been referred to the difference between the wall temperature and the mean bed temperature at that point. The contact resistance at the wall has been taken as negligible, i.e., the temperature of the bed at $y = 0$ is taken as equal to the wall temperature. The justification for neglecting the thermal resistance for a moving bed has been discussed previously [1].* Substitution from (5) and (7) into (12) gives

$$\mathrm{Nu} = \mathrm{Ki}_q \left\{ \frac{\mathrm{Ki}_q}{2} - \sum_{n=1}^{\infty} A_n \exp(-\mu_n^2 X)\,[(\nu_2^2 - 1)\,b_2(\mu_n) - \right.$$

*It has been shown [1] that the contact resistance becomes negligibly small in comparison with the heat-transfer resistance in the bed at a sufficient distance from the inlet section.

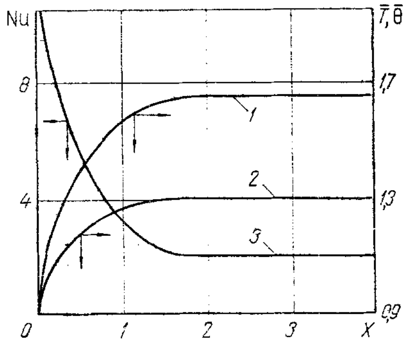

Fig. 3

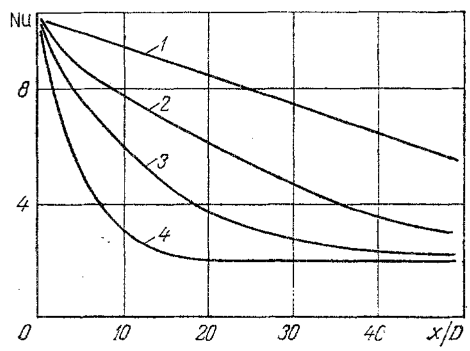

Fig. 4

Fig. 3. Variation in mean temperature (1), potential (2), and heat-transfer rate (3) with the reduced length.

Fig. 4. Variation in heat-transfer rate along the channel: 1) Pe = 100; 2) 50; 3) 25; 4) 10.

$$-(\nu_1^2-1)\,b_1(\mu_n)]+\sum_{n=1}^{\infty}A_n\exp(-\mu_n^2X)\left[(\nu_2^2-1)\,b_2(\mu_n)\times\right.$$
$$\left.\times\frac{\sin(\nu_1\mu_n)}{\nu_1\mu_n}-(\nu_1^2-1)\,b_1(\mu_n)\,\frac{\sin(\nu_2\mu_n)}{\nu_2\mu_n}\right]\Bigg\}^{-1}. \qquad (13)$$

The heat-transfer rate for a moving bed is dependent on the speed, the thermophysical characteristics, and the mass-transfer characteristics, as well as the channel size and the phase-transition criterion. The Nusselt number decreases as the reduced length increases for the initial part of a channel, but at $X > X_{st}$ it reaches its lower limit, after which the infinite sum in (13) becomes negligible.

Consequently, heat transfer complicated by mass transfer also shows thermal stabilization over the initial section. The rate of change in the Nusselt number is dependent on the quantities that characterize the mass transfer (Lu, Pn, Ko, ε) under otherwise equal conditions. In the stabilized region (for $X > X_{st}$) we obtain that (13) becomes

$$\mathrm{Nu}_{\min}=2=\mathrm{const}, \qquad (14)$$

$$\alpha_{\min}=2\lambda_q/D. \qquad (15)$$

In this region, the heat transfer is determined only by the thermal resistance arising from the thermal conductivity of the bed in the channel; the speed and the mass-transfer characteristics have no effect.

The effects of the major factors on the coupled heat and mass transfer in drying in a moving bed have been examined by computation with a BÉSM-4 computer for widely varying conditions. Figures 2-4 show calculations for the following values of the parameters: $10 \le \mathrm{Pe} \le 100$; $0 \le x/D \le 50$; $0 \le X \le 4$; $\mathrm{Ki}_q = 0.9$; $\mathrm{Bi}_q = 5$; $\mathrm{Bi}_m = 1$; Ly = 0.4; Pn = 0.6; Ko = 5; and $\varepsilon = 0.4$. If $X \ge 0.8$, the temperature and potential have linear distributions over the cross section. Essentially constant mean temperature and potential occur for $X \ge 1.8$. Thermal stabilization occurs within about the same reduced length (Fig. 3). Therefore, the thermal stabilization length is given approximately by the following for these conditions:

$$X_{st}\simeq 1.8;\quad (l/D)_{st}\simeq 1.8\,\mathrm{Pe}. \qquad (16)$$

NOTATION

a_q, effective thermal diffusivity; a_m, diffusion coefficient in bed; c_q, specific heat of material; c_m, specific isothermal mass capacity; D, bed thickness; q_w, heat-flux density at heated wall; t, temperature; l, thermal-stabilization length; $u = c_m\Theta$, moisture content; x, y, longitudinal and transferse coordinates; r, latent heat of evaporation; v, bed velocity; α, heat-transfer coefficient; β, mass-transfer coefficient; δ, thermal-gradient coefficient; Θ, moisture-transfer potential; λ_q, effective thermal conductivity; λ_m, mass conductivity;

τ, time; $Bi_q = \alpha_f D/\lambda_q$; $Bi_m = \beta D/\lambda_m$, Biot number; ε, phase-transition criterion; $Ki_q = q_w D/\lambda_q(t_0 - t_f)$, Kirpichev number; $Ko = rc_m(\Theta_0 - \Theta_f)/c_q(t_f - t_0)$, Kossovich number; $Ly = a_m/a_q$, Lykov number; $Nu = \alpha D/\lambda_q$, Nusselt number; $Pe = vD/a_q$, Peclet number; $Pn = \delta(t_f - t_0)/c_m(\Theta_0 - \Theta_f)$, Posnov number; $T = (t_0 - t)/(t_0 - t_f)$, temperature; $X = (1/Pe)\cdot(x/D)$, reduced length; $Y = y/d$, coordinate; $\Theta^* = (\Theta_0 - \Theta)/(\Theta_0 - \Theta_f)$, moisture-transfer potential. Indices: 0, inlet cross section; st, stabilization; f, medium around free bed surface; m, mass transfer; q, thermal; w, at heating-plate surface.

LITERATURE CITED

1. V. A. Kalender'yan and V. V. Kornaraki, Methods of Accelerating the Heat Transfer to a Dense Moving Bed [in Russian], Vishcha Shkola, Kiev (1973).
2. V. V. Kornaraki, Nauchn.-Tekh. Ref. Sb. Abrazivy, No. 6, 9-14 (1975).
3. A. V. Lykov and Yu. A. Mikhailov, The Theory of Heat and Mass Transfer [in Russian], Gosenergoizdat, Moscow–Leningrad (1963).
4. M. I. Makovozov, Proceedings of the Moscow Scientific-Research Institute of Meat and Dairy Products [in Russian], No. 8, Moscow (1958), p. 82.
5. S. Bruin, Int. J. Jeat Mass Transfer, 12, 45-59 (1969).
6. M. D. Mikhailov, Int. J. Heat Mass Transfer, 16, 2155-2164 (1973).
7. M. D. Mikhailov and B. K. Shishedjiev, Int. J. Heat Mass Transfer, 18, 15-24 (1975).